Problem Set 8

Problems to Computational Astrophysics, WS 2013/2014

Prof. Dr. Friedrich Röpke, Prof. Dr. Christian Klingenberg, Sebastian Ohlmann
Offices: Campus Hubland Nord, 31.01.017, 30.02.012, 31.01.003

Hand in until Monday, 20.01.2014, 12.00 pm
Tutorial on Tuesday, 21.01.2014, 10.15 am

1. Fast Fourier Transform

a) (P) To see how Fast Fourier Transform works, calculate "by hand" the discret Fourier transform F_n of the (discretized) function $f_k = [2, 5, -7, 2, 3, -1, 9, 0]$. Follow the steps of the FFT algorithm in the notation and convention for the DFT used in class.

b) (P) Write a pseudocode for the algorithm.

c) (H) Implement this pseudocode. Apply it to the values from a) and compare to your previous result.

d) (H) Test the scaling of your implementation, i.e. compare the run times for different input sizes. Does it show the expected behavior?

2. Relaxation methods for solving the Poisson equation (P)

Download the program "Poisson" from the course's web page and run it. Play with the different relaxation methods and run the demos for multigrid and Gauss-Seidel.
Note: Contrary to what was said in class, running the same problem with Gauss-Seidel is much slower than running it with Jacobi's method (the run time is shown at the end of the calculation in the terminal, not in the GUI). This, however, is a problem of our particular implementation in *Python* (there may be better ways to implement it...).

3. Shock formation (P)

Consider the equation $u_t + (u^2)_x = 0$ with smooth initial data $u_0(x)$ for which $u_0'(x)$ is somewhere negative. Show that the "wave" will break at time

$$T = \frac{-1}{\min 2u_0'(x)},$$

meaning that a shock will built up. Generalize this for $u_t + f(u)_x = 0$ where $f(u)$ is a convex function of u.

4. Lagrangian coordinates (P)

Consider the equations for isentropic compressible gas dynamics in one spatial dimension:

$$
\begin{aligned}
\rho_t + (\rho u)_x &= 0 \\
(\rho u)_t + \Big[\rho u^2 + p(\rho)\Big]_x &= 0
\end{aligned}
$$

Reformulate the equations using Lagrangian coordinates.
Note: You have to determine the mapping $X(\xi, t)$ that denotes the position of particle ξ at time t. The relation between the velocity in Lagrangian coordinates U and the velocity in Eulerian coordinates u is then $U(\xi, t) = u(X(\xi, t), t)$.

5. Wave equation (P)

Consider the equations of Exercise 4 with $p(\rho) = A\rho^\gamma$ ($A > 1$ and $\gamma > 1$, both const.), a given constant state (ρ_0, u_0) and a small perturbation that is superimposed to the solution (the square of the perturbation is negligible). Derive a linearized equation for the propagation of this perturbation. The result is a wave equation with propagation speed c. What is the formula for c?

Exercises marked with (P) have to be presented in the exercise, those marked with (H) have to be handed in. Programs can be sent per e-mail to `sohlmann@astro.uni-wuerzburg.de`.