Problem Set 1

Problems to Theoretical Astrophysics, SS 2014

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Tutorial on Tuesday, 29.04.2014, 10.15 am

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1. Core collapse supernova

In a core-collapse supernova, the iron core of a massive star reaches the Chandrasekhar mass, renders instable, and collapses to a neutron star. This supernova explosion is driven by the released gravitational energy. Estimate the order of magnitude of the released gravitational binding energy (cgs unit, i.e. energy in erg).

2. Validity of the Newtonian approximation

Although the validity of the Newtonian approximation can only be judged using the more fundamental theory of General Relativity, we will derive a simple estimate in the following.
The gravitational potential Φ has units

$$[\Phi] = \left(\frac{\mathrm{cm}}{\mathrm{s}}\right)^2,$$

i.e. velocity squared. Taking into account the fact that nothing moves faster than the speed of light in vacuum, c, this value is bounded by c^2. Thus, the Newtonian theory is a good approximation as long as

$$|\Phi| \ll c^2 \tag{1}$$

is fulfilled.

a) Suppose a particle with mass M is given at the origin of the coordinate system. What is its gravitational potential? The absolute value of the potential can be equated with the square of a certain velocity. Which special velocity is meant here? Derive a critical radius from condition (1) such that the Newtonian approximation is valid if the radius is much larger than this critical value.

b) The well-known *Schwarzschild radius* is twice as large as this radius; the factor 2 can be derived only in the full general relativistic treatment. Compute the Schwarzschild radius for the Earth, the Sun, and for a neutron star. At which conclusions do you arrive?

3. The Plummer sphere

The *Plummer sphere* was introduced in 1911 by English astronomer H. C. Plummer as a model for explaining observations of globular clusters. Its potential is given by

$$\Phi_\mathrm{P} = -\frac{GM}{\sqrt{r^2 + a_\mathrm{P}^2}}.$$

If the Plummer radius a_P is small compared to the radius r, this potential tends to the gravitational potential of a point mass.

a) What is the density profile of the Plummer sphere? Compute $\rho(r)$ and sketch or plot the profile.

b) Compute the mass $M(r)$ enclosed in the radius r.

4. The Sun in our Galaxy

a) The sun revolves around the center of the Galaxy on a nearly circular orbit with a distance of $R_0 \approx 8\,\mathrm{kpc}$ and a velocity of $v_0 \approx 200\,\mathrm{km\,s^{-1}}$. Compute the total mass of the Galaxy assuming that the mass is concentrated at the center. Show that the escape velocity near the Sun is $v_\mathrm{e} = \sqrt{2}v_0$.

b) Since local stars are observed with velocities up to $500\,\mathrm{km\,s^{-1}}$, the model of a point mass does not fit very well the mass distribution of the Galaxy. Therefore, we model the Galaxy as a Plummer sphere with $a_\mathrm{P} = R_0$. What is the total mass? How much of this mass is located outside of the solar orbit? Show that the high velocities of local stars cannot be explained in this model. What do you conclude?