Problem Set 3

Problems to Theoretical Astrophysics, SS 2014
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Tutorial on Tuesday, 13.05.2014, 10.15 am

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1. Stability of the Lagrangian points L_4 and L_5 *(optional)*

Install SciPy from `http://www.scipy.org/Installing_SciPy` and download the program `Lagrange.tar.gz` from `http://astro.uni-wuerzburg.de/~sohlmann/teaching`. After unzipping, start the program in the folder `Lagrange` with the command `python gui.py`. Then, a graphical user interface will appear with different options. Try to position the test masses in stable orbits for different masses (in solar masses) and distances (in astronomical units). Choose mass ratios larger and smaller than the critical value for the stability of L_4 and L_5. What do you observe? Test for values which correspond to the Sun-Earth system, the Earth-Moon system, and the Sun-Jupiter system.

2. Globular cluster

Consider an isolated globular cluster in equilibrium (thus obeying the virial theorem). It consists of stars and interstellar material (ISM). Let the initial radius be R_i and let the part f_* of the initial mass M_i consist of stars. Thus, the initial mass of the ISM is $(1-f_*)M_\mathrm{i}$. Assume that stars and ISM have the same velocity distribution and the same spatial distribution.
Massive stars evolve fastest and die as supernova explosions. These explosions remove all ISM from the cluster. Because only a small part of the stars is massive, neither the total mass of the stars nor their velocity distribution or spatial distribution changes.

a) For which values of f_* the globular cluster remains bound when the gas is expelled (you may assume constant density)?

b) Assume that the globular cluster remains bound and virializes again (preserving the total energy). How does the radius change? Find an expression for the final radius R_f in terms of the initial radius R_i. How does this expression behave for $f_* \to 1$ and for the critical value of a bound system?

3. Relations between thermodynamic variables

The relation

$$\mathrm{d}E = \frac{V}{N}\left(\frac{\partial e}{\partial s}\right)_n \mathrm{d}S + \left[\frac{E}{V} - \frac{N}{V}\left(\frac{\partial e}{\partial n}\right)_s\right]\mathrm{d}V + \left[\left(\frac{\partial e}{\partial n}\right)_s - \left(\frac{\partial e}{\partial s}\right)_n \frac{VS}{N^2}\right]\mathrm{d}N$$

was used in the lecture. Show how it follows from

$$\mathrm{d}e = \left(\frac{\partial e}{\partial n}\right)_s \mathrm{d}n + \left(\frac{\partial e}{\partial s}\right)_n \mathrm{d}s.$$

4. Heat capacities

Show that the heat capacities at constant volume (C_V) and constant pressure (C_P) are related by

$$C_P - C_V = \frac{PV}{T}\frac{\chi_T^2}{\chi_\rho} \tag{1}$$

and

$$\frac{C_P}{C_V} \equiv \gamma = 1 + \frac{PV}{TC_V}\frac{\chi_T^2}{\chi_\rho}. \tag{2}$$

a) Take the definitions $C_V = T(\partial S/\partial T)_V$ and $C_P = T(\partial S/\partial T)_P$ as a starting point. Use the total differentials of $S(T,P)$ and $P(T,V)$ to show

$$C_P - C_V = -T\left(\frac{\partial S}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V.$$

b) Use the Maxwell relation for Gibbs energy $G(T,P)$,

$$\frac{\partial^2 G}{\partial T \partial P} = \frac{\partial^2 G}{\partial P \partial T},$$

to show

$$C_P - C_V = -T\frac{\left(\frac{\partial P}{\partial T}\right)_V^2}{\left(\frac{\partial P}{\partial V}\right)_T}.$$

c) Use this to show relations (1) and (2).

Formulae:

$$\gamma = \frac{C_p}{C_v}$$

$$\chi_\rho = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_T$$

$$\chi_T = \left(\frac{\partial \ln P}{\partial \ln T}\right)_\rho$$