Problem Set 8

Problems to Theoretical Astrophysics, SS 2014

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Tutorial on Tuesday, 01.07.2014, 10.15 am

1. Gravitational energy of a polytrope

Show that the gravitational energy of a star described as a polytrope with index n is given by

$$E_\mathrm{g} = -\frac{3}{5-n}\frac{GM^2}{R},$$

where M is the mass and R the radius of the star and G the gravitational constant.
Hint: Show first that

$$\frac{\mathrm{d}P}{\rho} = (n+1)\,\mathrm{d}\left(\frac{P}{\rho}\right)$$

from the polytropic relation introduced in the lecture.
Partial integration of the gravitational energy yields

$$E_\mathrm{g} \equiv -G\int_{\text{Center}}^{\text{Surface}} \frac{m}{r}\,\mathrm{d}m = -\frac{GM^2}{2R} - \frac{G}{2}\int_{\text{Center}}^{\text{Surface}} \frac{m^2}{r^2}\,\mathrm{d}r.$$

2. Extremely relativistic degenerate star in hydrostatic equilibrium

a) Show that a extremely relativistic degenerate star in hydrostatic equilibrium is unbound, i.e. the total energy is given by $E_\mathrm{tot} = E_\mathrm{g} + E_\mathrm{i} = 0$.

b) How large is the internal energy?

Hint: Derive first the virial theorem for the generalized equation of state

$$\zeta u = 3\frac{P}{\rho},$$

where ζ is assumed to be constant throughout the star.