Problem Set 2

Problems to Computational Astrophysics, WS 2013/2014

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Hand in until Monday, 04.11.2013, 12.00 pm
Tutorial on Tuesday, 05.11.2013, 10.15 am

1. Finite differences

a) **(H)** Write a program in any programming langauage of your choice computing the first derivative $f'(x)$ of a function $f(x)$ using the forward, backward, and central difference formulae for a given step size h.

b) **(H)** Compute $f'(0)$ for $f(x) = \exp(x)$ for different step sizes. Plot the relative error over the step size (choose a logarithmic scale for both axes; use step sizes down to $h = 10^{-15}$).

c) **(P)** What do you observe for very small step sizes? Explain the behaviour of the relative error.

2. Lax-Friedrichs method

a) **(P)** Consider $u_t + a\, u_x = 0$ for $u(x,t) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $a > 0$. The Lax-Friedrichs method is defined by

$$U_j^{n+1} = \frac{1}{2}\left(U_{j-1}^n + U_{j+1}^n\right) - \frac{\Delta t}{2\Delta x} a \left(U_{j+1}^n - U_{j-1}^n\right).$$

Use a Taylor expansion to compute the error to first non-vanishing order. Which equation is solved by the Lax-Friedrichs method when retaining these terms?

b) **(H)** Implement the Lax-Friedrichs scheme in any programming language of your choice for $u_t + u_x = 0$ and apply it to the initial data

$$u(x,0) = \begin{cases} 1 & \text{if } x < 0 \\ 0 & \text{if } x > 0. \end{cases}$$

What do you observe?

Exercises marked with (P) have to be presented in the exercise, those marked with (H) have to be handed in. Programs can be sent per e-mail to `sohlmann@astro.uni-wuerzburg.de`.