Problem Set 5

Problems to Computational Astrophysics, WS 2013/2014

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Hand in until Monday, 02.12.2013, 12.00 pm
Tutorial on Tuesday, 03.12.2013, 10.15 am

1. Butcher tableaux for Runge-Kutta schemes (P)

As discussed in class, there is a wide variety of Runge-Kutta schemes that is characterized by different choices of the coefficients b_j, c_j and a_{jl}. A nice way of representing these coefficients is Butchers tableau:

$$\begin{array}{c|ccccc} c_1 & & & & & \\ c_2 & a_{21} & & & & \\ c_3 & a_{31} & a_{32} & & & \\ \vdots & & & & & \\ c_s & a_{s1} & a_{s2} & \cdots & a_{s,s-1} & \\ \hline & b_1 & b_2 & \cdots & b_{s-1} & b_s \end{array}$$

a) What are the corresponding tableaux for Euler's method, the midpoint integration, and RK4 discussed in class?

b) Can the coefficients be chosen arbitrarily? What are the requirements for a consistent choice of b_j coefficients in the Butcher tableau?

c) For the three-stage Runge-Kutta method RK3 (also sometimes called RK32) the Butcher tableau reads

$$\begin{array}{c|ccc} 0 & & & \\ 1/2 & 1/2 & & \\ 1 & -1 & 2 & \\ \hline & 1/6 & 2/3 & 1/6 \end{array}$$

What are the corresponding equations of this method? Explain how it works.

2. Sun-earth system (H)

Program the orbit of the earth around the sun using the Euler method and the RK4 scheme for numerical integration of the trajectory. Chose $\Delta t = 1\,\mathrm{d}$ as step size and calculate the error in the total energy after one orbit of the earth around the sun. How do the methods compare?

Exercises marked with (P) have to be presented in the exercise, those marked with (H) have to be handed in. Programs can be sent per e-mail to `sohlmann@astro.uni-wuerzburg.de`.