Problem Set 6

Problems to Computational Astrophysics, WS 2013/2014

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Hand in until Monday, 09.12.2013, 12.00 pm
Tutorial on Tuesday, 10.12.2013, 10.15 am

1. Stellar encounters (P)

a) In class, it was mentioned that stellar encounters can be neglected when modeling a galaxy as a collection of stars, that is, it can be treated as an essentially collisionless system. Give a reason for why very close star-star encounters are rare.

   Hint: As a simple galaxy model assume N stars uniformly distributed within a region of radius R. For a significant change in the velocity v of a star, it must come close enough to another star of mass M (take $1M_\odot$ as a typical value) – but how close is close enough? Chose the distance d that would correspond to the radius of a circular orbit with orbital velocity equal to v around the star. With this value of d, we define a "cross section" for stellar encounters. Let D define the mean free path of a star in a galaxy. It implies a "relaxation time" t_r – the time it travels freely without encounters.

b) Give approximations for D and t_r for the Milky Way (what are typical values of N, R, and v here)? How does t_r compare to the age of the Milky Way?

c) Now calculate the same for a globular cluster of stars that contains about 5×10^5 stars, has a median radius of about 10pc and a central velocity dispersion of $\sim$7 km s^{-1}. Compare the result to its lifetime of $\sim 10^{10}$ years.

   Note: The result seems to contradict the statement made in class that galaxies can be treated as essentially collisionless systems whereas stellar clusters cannot. However, our argument developed here is somewhat simplistic as it considers only strong two body encounters. A rigorous treatment (such as provided in Binney and Tremaine's book "Galactic Dynamics") would also estimate the cumulative effect of many weak encounters and things look different then.

2. PM-scheme for isolated systems (P)

The particle-mesh scheme for solving the many-body problem is suitable for periodic boundaries. If we apply it to isolated systems, we encounter problems that can be solved by a trick called *zero-padding*. We explore this here:
Download the example of a particle-mesh code called "PM" from the course's web page `www.mpa-garching.mpg.de/~fritz/teaching/notes`
(login: student, password: wuerzburg) and run it by starting the graphical user interface with the command `python gui.py` from the `src` subdirectory (you need *Python*

and the *SciPy* (`www.scipy.org`) package for it; both are freely available for different operating systems).

a) Check the box *radial distribution*. This produces an initial setup by distributing the particles randomly inside a circle. Now run the program. What do you observe? Does this correspond to the expectation of how an isolated spherical particle distribution should behave? If not, what is the problem?

b) Now try again setting *zero padding* to a value of 1.0. What does it do? Does this help? If so, why? Now, play with different values for the zero-padding. What values give acceptable results?

Exercises marked with (P) have to be presented in the exercise, those marked with (H) have to be handed in. Programs can be sent per e-mail to `sohlmann@astro.uni-wuerzburg.de`.