Problem Set 9

Problems to Computational Astrophysics, WS 2013/2014

Prof. Dr. Friedrich Röpke, Prof. Dr. Christian Klingenberg, Sebastian Ohlmann
Offices: Campus Hubland Nord, 31.01.017, 30.02.012, 31.01.003

Hand in until Monday, 27.01.2014, 12.00 pm
Tutorial on Tuesday, 28.01.2014, 10.15 am

---

1. Riemann problem (H)

Program the solution of the following initial value problems using the Lax-Friedrich as well as the Lax-Wendroff method:

a)
$$u_t + u_x = 0,$$

b)
$$u_t + (u^2)_x = 0,$$

c)
$$u_t + (u^3)_x = 0.$$

Use in each case the Riemann initial data

I.
$$u = \begin{cases} -1 & \text{if } x < 0 \\ +3 & \text{if } x > 0, \end{cases}$$

II.
$$u = \begin{cases} +3 & \text{if } x < 0 \\ -1 & \text{if } x > 0. \end{cases}$$

Refine the resolution to obtain more exact solutions. What do you observe?
Note: The Lax-Friedrich method for the scalar equation $u_t + f(u)_x$ can be stated as follows:

$$U_j^{n+1} = \frac{1}{2}\left(U_{j-1}^n + U_{j+1}^n\right) - \frac{\Delta t}{2\Delta x}\left[f\left(U_{j+1}^n\right) - f\left(U_{j-1}^n\right)\right].$$

The two step method after Lax-Wendroff for the scalar equation $u_t + f(u)_x$ is defined as:

$$\begin{aligned} U_{j+\frac{1}{2}}^{n+\frac{1}{2}} &= \frac{1}{2}\left(U_j^n + U_{j+1}^n\right) - \frac{\Delta t}{2\Delta x}\left[f\left(U_{j+1}^n\right) - f\left(U_j^n\right)\right], \\ U_j^{n+1} &= U_j^n - \frac{\Delta t}{\Delta x}\left[f\left(U_{j+\frac{1}{2}}^{n+\frac{1}{2}}\right) - f\left(U_{j-\frac{1}{2}}^{n+\frac{1}{2}}\right)\right]. \end{aligned}$$

2. Riemann problem for the Euler equations (P)

Download the program "riemann_classic" from the course's web page and run it. Play with different initial conditions. Note that there are predefined initial conditions that give rise to specific flow patterns (double-click on them to choose the corresponding values), but you can also enter your choice of values directly.

Exercises marked with (P) have to be presented in the exercise, those marked with (H) have to be handed in. Programs can be sent per e-mail to `sohlmann@astro.uni-wuerzburg.de`.