

# Problem Set 6

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## Problems to Theoretical Astrophysics, SS 2014

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### 1. Equilibrium condition for gas mixtures

In the lecture, the equilibrium condition

$$\sum_i \mu_i dY_i = 0 \quad (1)$$

was obtained assuming a thermally isolated system with constant volume.

- a) Consider the general case of a non-isolated system being able to perform work. Show that

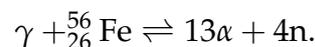
$$d\left(\frac{\varepsilon}{n}\right) + Pd\left(\frac{1}{n}\right) \leq Tds.$$

- b) Give reasons why the equilibrium condition (1) holds if  $n$  and  $s$  are fixed.  
c) Show that the equilibrium condition also holds for a system at constant pressure and constant temperature.

*Hint:* Use the Gibbs free energy  $G$ .

### 2. Photodisintegration of iron

In the course of core collapse supernovae, but also thermonuclear supernovae, thermodynamic states occur that allow iron group nuclei to disintegrate. Consider the photodisintegration of  ${}^{56}_{26}\text{Fe}$ ,



- a) Compute the energy  $Q$  necessary for the dissociation.  
b) What is the *Saha equation* for the photodisintegration assuming the statistical weights  $g_\alpha = 1$ ,  $g_n = 2$ , and  $g_{\text{Fe}} \simeq 1.4$ ?

*Hint:* The masses of the species can be approximated here (but not when computing  $Q$ !) by the product of the number of nucleons and the atomic mass unit,  $m(A) \approx Am_{\text{u}}$ .

- c) Consider a state where the mass of the material consists half of  ${}^{56}_{26}\text{Fe}$  and half of  $\alpha$  particles ("50% dissociation"). Derive the relation

$$\log_{10} \rho' = a + b \log_{10} T_9 - \frac{c}{T_9}$$

between density  $\rho' = \rho / (\text{g cm}^{-3})$  and temperature  $T_9 = T / (10^9 \text{ K})$  and determine the constants  $a$ ,  $b$ , and  $c$ .

*Hint:* Take into account

$$n_n = \frac{4}{13} n_\alpha,$$

following from the reaction equation.