Problem Set 6

Problems to Theoretical Astrophysics, SS 2014

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Tutorial on Friday, 20.06.2014, 12.30 pm

1. Equilibrium condition for gas mixtures

In the lecture, the equilibrium condition

$$\sum_i \mu_i \, \mathrm{d}Y_i = 0 \tag{1}$$

was obtained assuming a thermally isolated system with constant volume.

a) Consider the general case of a non-isolated system being able to perform work. Show that

$$\mathrm{d}\left(\frac{\varepsilon}{n}\right) + P\mathrm{d}\left(\frac{1}{n}\right) \leq T\mathrm{d}s.$$

b) Give reasons why the equilibrium condition (1) holds if n and s are fixed.

c) Show that the equilibrium condition also holds for a system at constant pressure and constant temperature.

Hint: Use the Gibbs free energy G.

2. Photodisintegration of iron

In the course of core collapse supernovae, but also thermonuclear supernovae, thermodynamic states occur that allow iron group nuclei to disintegrate. Consider the photodisintegration of ${}^{56}_{26}\mathrm{Fe}$,

$$\gamma + {}^{56}_{26}\,\mathrm{Fe} \rightleftharpoons 13\alpha + 4\mathrm{n}.$$

a) Compute the energy Q necessary for the dissociation.

b) What is the *Saha equation* for the photodisintegration assuming the statistical weights $g_\alpha = 1$, $g_\mathrm{n} = 2$, and $g_\mathrm{Fe} \simeq 1.4$?

Hint: The masses of the species can be approximated here (but not when computing Q!) by the product of the number of nucleons and the atomic mass unit, $m(A) \approx Am_\mathrm{u}$.

c) Consider a state where the mass of the material consists half of $^{56}_{26}\mathrm{Fe}$ and half of α particles ("50% dissociation"). Derive the relation

$$\log_{10}\rho' = a + b\log_{10} T_9 - \frac{c}{T_9}$$

between density $\rho' = \rho/(\mathrm{g\,cm^{-3}})$ and temperature $T_9 = T/(10^9\,\mathrm{K})$ and determine the constants a, b, and c.

Hint: Take into account

$$n_\mathrm{n} = \frac{4}{13} n_\alpha,$$

following from the reaction equation.